

Is the first excited state of the α -particle a breathing mode?

Sonia Bacca^{1,2}, Nir Barnea³, Winfried Leidemann^{4,5}, Giuseppina Orlandini^{4,5}

¹*TRIUMF, 4004 Wesbrook Mall, Vancouver, B.C. V6J 2A3, Canada*

²*Department of Physics and Astronomy,*

University of Manitoba, Winnipeg, MB, R3T 2N2, Canada

³*Racah Institute of Physics, The Hebrew University, 91904, Jerusalem, Israel*

⁴*Dipartimento di Fisica, Università di Trento, I-38123 Trento, Italy*

⁵*Istituto Nazionale di Fisica Nucleare, TIFPA, I-38123 Trento, Italy*

Abstract

The isoscalar monopole excitation of ${}^4\text{He}$ is studied within a few-body *ab initio* approach. We consider the transition density to the low-lying and narrow 0^+ resonance, as well as various sum rules and the strength energy distribution itself at different momentum transfers q . Realistic nuclear forces of chiral and phenomenological nature are employed. Various indications for a collective breathing mode are found: i) the specific shape of the transition density, ii) the high degree of exhaustion of the non-energy-weighted sum rule at low q and iii) the complete dominance of the resonance peak in the excitation spectrum. For the incompressibility K of the α -particle values between 20 and 30 MeV are found.

PACS numbers: 24.30.Cz, 21.10.Re, 25.30.Fj, 27.10.+h, 25.55

The quantum breathing mode (monopole oscillation) is the object of continuous theoretical and experimental investigations in a large variety of systems as nuclei and trapped nanoplasmas or cold atoms (see e.g. [1, 2] and references therein). In fact it appears as one of the most important properties that allow to diagnose the underlying force. The isoscalar giant monopole resonance (ISGMR), also known as nuclear breathing mode, is one of the collective nuclear excitations that are well established and much discussed in heavier nuclei, also because of possible interesting relations to the nuclear matter incompressibility (see e.g. [3, 4] and references therein).

The lightest nucleus where the breathing mode has been discussed is the α -particle. The discussion has a rather long history. It was triggered by the results of an inclusive electron scattering experiment on ^4He in 1965 [5], where the transition form factor $|\text{F}_{\mathcal{M}}(q)|^2$ to the 0^+ resonance (0_R^+) had been measured for various momentum transfer q . The interpretation of the resonance as a collective breathing mode was suggested a year later [6]. A fair agreement both for the excitation energy E_R and $|\text{F}_{\mathcal{M}}(q)|^2$ was obtained. In the following years the collectivity of the resonance was object of further discussion.

The intent of the present work is to analyze the collectivity issue of the 0_R^+ from a modern few-body *ab initio* point of view [7], as an interesting bridge between few- and many-body physics. Further below it will become evident that various results of our *ab initio* few-body calculation point to a breathing mode interpretation of the resonance.

As an introduction, we give a summary of the discussion on the 0_R^+ , however leaving out other theoretical work where the issue of collectivity is not addressed explicitly. After the work of [5, 6] mentioned above, in 1970 the 0_R^+ was the object of an electron scattering experiment at lower q [8], and in the following decade the generator coordinate method [9, 10] was applied to investigate the resonance [11, 12]. Within this method collective motions are studied microscopically, introducing however a collective path. Again a fair agreement with experiment was found. The results gave *“further evidence that the first excited state in ^4He can be interpreted as a compressional monopole state”* [12]. A translation invariant shell model calculation, which included two-particle two-hole (2p-2h) configurations, was carried out in 1981 [13] and there it was concluded that the comparison of the obtained results with the experimental $|\text{F}_{\mathcal{M}}(q)|^2$ *“casts doubts on the usual breathing mode interpretation”*. A similar conclusion was drawn in 1986 [14] by comparing the results of a two- and four-particle excitation model to data that included also new results at higher q [15]. There

it was stated that the resonance “*has very little of the breathing mode in it and consists basically of more complex excitations*”. In 1987 two microscopic models were applied [16]: one in terms of a collective variable, the other in terms of a cluster variable (similar to the resonating group approach). It was concluded that the 0_R^+ “*has a cluster character*” (3+1). A year later it is stated [17] that “*these two hypotheses are not mutually exclusive*”. In fact it was already shown in [6] that in a translation invariant harmonic oscillator model, where one nucleon is excited from the $0s$ to the $1s$ shell, the transition density changes sign at the ^4He radius. This is precisely the form of the excess density of a breathing mode, though the model may appear as non-collective, because of its interpretation as mean field or (3+1) cluster. This common feature is not very surprising in a s -shell scenario, since the transition density for $q \rightarrow 0$ can be written as a function of just the hyperradius $\rho^2 = \sum_i^4 r_i^2$, one of the six *collective* coordinates defined by the group $GL^+(3, R)$ [18, 19]. At the end of the 1980s the 0_R^+ was understood as “*a superposition of simple $1p$ - $1h$ excitations and not as a collective state*” [20].

We observe that in all these early work the criterion for the collectivity of the 0_R^+ has been mainly the agreement with experimental data: if in a collective (non collective, i.e. purely mean field based) model the data could be described sufficiently well, it was simply concluded that the 0_R^+ has a collective (non-collective) character.

No further insight in the issue of collectivity came up till 2004, when it was reconsidered within a few-body *ab initio* approach [21]. Using a truncated version of a realistic nucleon-nucleon (NN) potential, Argonne V8’ (AV8’) [22], and a phenomenological three-nucleon force (3NF), it was concluded with a sum rule argument that the 0_R^+ is not a breathing mode. Another motivation against the collectivity was the large overlap of the 0_R^+ wave function with the trinucleon ground state. However, the density is an integral property, therefore it is perfectly possible, as already indicated in [6, 17], that a large overlap of the 0_R^+ state with a (3+1) cluster gives rise to the density of a breathing mode. The sum rule arguments are questioned below.

For many-body systems there is a general consensus that the breathing mode collectivity is signaled by two typical features: (i) the above mentioned peculiar form of the transition density; (ii) the degree of exhaustion of the energy weighted sum rule [23] by the resonance strength. For ^4He the latter was often examined, while the former was considered (besides in [6]) only in [21], where it was presented, but not commented.

As already pointed out, the aim of the present work is the study of collectivity from a modern *ab initio* few-body point of view. We will investigate whether an *ab initio* calculation of the ^4He inelastic isoscalar monopole (InISM) strength exhibits features that are believed to characterize a collective behavior in many-body systems, and how they depend on different nuclear forces. We will use two realistic potential models including the Coulomb force: the chiral effective field theory NN potential at next-to-next-to-next-to leading order (N^3LO) [24] plus the N^2LO 3NF [25] and the AV18 NN potential [26] plus the UIX 3NF [27] (both NN potentials lead to excellent fits to NN scattering data). In this work we will consider both the form of the transition density to the 0_R^+ and the degree of exhaustion by the resonance strength of various SRs. We will investigate to what extent these features appear and conclude accordingly whether realistic nuclear forces, the only input of a four-body *ab initio* calculation, allows to picture the excitation as collective. In addition we show that the strength distribution itself leads to much better insights.

Formalism. The InISM strength distribution is given by

$$S_{\mathcal{M}}(q, \omega) = \sum_{n=0}^{\infty} \langle n | \mathcal{M}(q) | 0 \rangle^2 \delta(\omega - E_n + E_0), \quad (1)$$

where ω is the excitation energy, and $|0\rangle, |n\rangle$ and E_0, E_n are eigenfunctions and eigenvalues of the nuclear Hamiltonian H , respectively (for continuum energies the sum is replaced by an integral). The InISM operator reads

$$\mathcal{M}(q) = \frac{1}{2} \left(\sum_{i=1}^A j_0(qr_i) - \langle 0 | \sum_{i=1}^A j_0(qr_i) | 0 \rangle \right), \quad (2)$$

where j_0 is the zero-th order spherical Bessel function (the dependence on the nucleon form factor is neglected).

In the low- q , long wavelength (LW), limit the InISM operator is proportional to

$$\mathcal{M}^{\text{LW}} = \frac{1}{2} \left(\sum_{i=1}^A r_i^2 - A \langle r^2 \rangle \right), \quad (3)$$

where $\langle r^2 \rangle$ is the mean square radius of the system. It is important to notice that \mathcal{M}^{LW} depends only on the collective variable ρ^2 , different from $\mathcal{M}(q)$.

Sum rules. As well explained in [28], sum rules “provide useful yardsticks for measuring quantitatively the degree of collectivity of a given excited state”. They are particular

expressions for the moments [29–31] defined as

$$m_n(q) = \int d\omega \omega^n S_{\mathcal{M}}(q, \omega). \quad (4)$$

The moment m_1 for the operator \mathcal{M}^{LW} is an interesting SR. Using the completeness property of the eigenstates of the Hamiltonian $H = T + V$, where T and V are the kinetic and potential energy operators, one finds

$$m_1 = \frac{1}{2} \langle 0 | [\mathcal{M}, [T + V, \mathcal{M}]] | 0 \rangle \equiv m_1(T) + m_1(V). \quad (5)$$

For local potentials m_1 coincides with $m_1(T)$ and in the LW limit one obtains

$$m_1^{\text{LW}}(T) = \frac{1}{2} \langle 0 | [\mathcal{M}^{\text{LW}}, [T, \mathcal{M}^{\text{LW}}]] | 0 \rangle = \frac{A}{2M} \langle r^2 \rangle, \quad (6)$$

where M is the nucleon mass and A the number of nucleons ($\hbar = c = 1$). Equation (6) is known as the Ferrell energy-weighted SR (FEWSR) [23].

For m_0 completeness leads to

$$m_0^{\text{LW}} = \langle 0 | \mathcal{M}^{\text{LW}} \mathcal{M}^{\text{LW}} | 0 \rangle. \quad (7)$$

While the FEWSR is considered “*model independent*”, m_0^{LW} is not. In fact the experimental value of $\langle r^2 \rangle$ (corrected for the nucleon finite size) can lead to a good estimate of the FEWSR. On the contrary for the evaluation of m_0^{LW} one would need one- and two-body ground-state densities. Particularly the latter is largely model dependent.

For the special case where all the transition strength is concentrated into one specific excited state the sum rules become very simple. Assuming that the breathing mode $|\text{BM}\rangle$ is such a state, it is evident that it exhausts 100% of the FEWSR [6]. In general, this assumption implies that for any n one has $m_n = (E_{\text{BM}} - E_0)^n |\langle \text{BM} | \mathcal{M}^{\text{LW}} | 0 \rangle|^2$ and therefore this state exhausts all sum rules completely:

$$r_n^{\text{LW}} = \frac{(E_{\text{BM}} - E_0)^n |\langle \text{BM} | \mathcal{M}^{\text{LW}} | 0 \rangle|^2}{m_n} = 1. \quad (8)$$

We note in passing that in [6] also a simple relation between the $|\text{F}_{\mathcal{M}}(q)|^2$ and the derivative of the elastic form factor was derived and that the same relation is found in [32] via the so-called “progenitor sum rule” [33], under the hypothesis of a unique collective state.

It is generally believed that the ratios r_n , and in particular r_1 , are good quantities to infer the degree of collectivity of a state. However, it is usually neglected that for $n = 1$

(and higher) one emphasizes the high-energy strength contribution. In fact, even in presence of a pronounced collective state, a negligible higher energy strength could lead to a rather small r_1 and thus to the wrong conclusion. In [28] it is made clear that the proper quantity to check is rather r_0 . It is the “*model independence*” of the FEWSR and the difficulty in calculating m_0 , especially for the heavier nuclei, that has led to the general attitude to consider r_1 . For ${}^4\text{He}$, however, we are able to evaluate m_0^{LW} accurately. The results for the moments m_0 and m_1 of $S_{\mathcal{M}}(q, \omega)$, reported in the following, are obtained via the Lanczos algorithm, analogously to what was done in [34] for the dipole operator. We perform our calculations by diagonalizing H on the hyperspherical harmonics (HH) basis up to sufficient convergence, using the effective interaction HH method [35, 36].

We test the accuracy of our results using the AV8’ potential [22]. Different from the other models this potential is *almost* non-local (the only non-local term, the spin-orbit term, does not contribute to $m_1^{\text{LW}}(V)$). Therefore we get an estimate of our numerical error via independent calculations of m_1^{LW} and of $\langle r^2 \rangle$ in Eq. (6). We find $m_1^{\text{LW}} = 183.43 \text{ fm}^4 \text{ MeV}$ to be compared to $183.62 \text{ fm}^4 \text{ MeV}$ from the FEWSR.

Results and discussion. We discuss our first criterion for collectivity, namely the specific shape of the transition density¹

$$\rho_{tr}(r) = |\langle 0_R^+ | \mathcal{M}^{\text{LW}} | 0 \rangle|^2. \quad (9)$$

In Fig. 1 we show $\rho_{tr}(r)$. One sees that the first criterion is met quite well since $\rho_{tr}(r)$ changes sign at a distance approximately equal to the root mean square radius of about 1.46 fm, for both our potential models. A similar behavior is found in [21] (see Fig. 2 there). Therefore we may conclude that this feature is rather independent on the force.

Next we study our second criterion for collectivity, namely sum rules. First we analyze r_0^{LW} , replacing in Eq.(8) $| \text{BM} \rangle$ by $| 0_R^+ \rangle$. We calculate the transition strength in the same way as we did in [37], but for \mathcal{M}^{LW} . Here we recall that it is the Lorentz integral transform method [38, 39] and a proper inversion algorithm that allows us to separate the resonant from the background contribution. For the chiral (phenomenological) potential we find for the transition strength the value of 3.53 (2.25) fm^4 and $m_0 = 6.80 \text{ (5.99) fm}^4$, leading to

¹ This definition would in principle involve an integral over the resonance region, but, as in other calculations in the literature for very narrow resonances, we approximate 0_R^+ by a bound state.

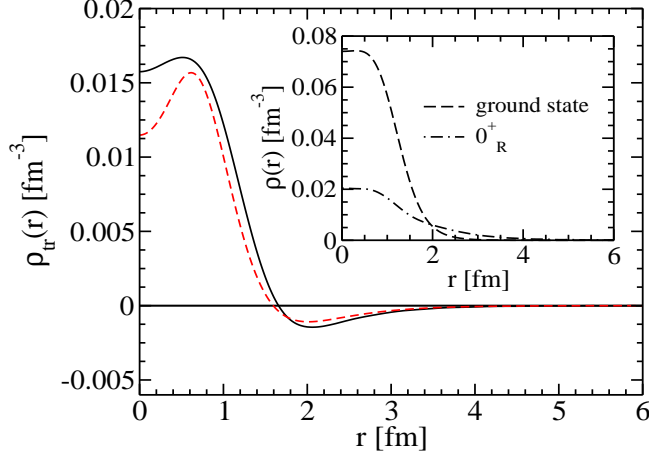


FIG. 1. (Color online) Transition density between the ground state and the 0_R^+ for the chiral (black full line) and the phenomenological (red dashed line) forces. In the inset the densities of the ground state and of the 0_R^+ for the chiral force are also shown.

$r_0^{\text{LW}} = 52\%$ (38%). Considering the observation by Rowe [28]: “A typical $T=0$ collective state exhausts something like 50 per cent” (of m_0), we are led to conclude that the chiral force generates a 0_R^+ of *collective* character.

Since in experiments the $|\mathcal{F}_{\mathcal{M}}(q)|^2$ is measured at finite momentum transfer, it is worth studying r_n at various q (in Eq. (8): $r_n^{\text{LW}} \rightarrow r_n(q)$, $\mathcal{M}^{\text{LW}} \rightarrow \mathcal{M}(q)$). Results are given in Table I. For both potential models $r_0(q)$ and $r_1(q)$ decrease with growing q , but differ somewhat. The low- q the results for r_0 are in line with those in the $q \rightarrow 0$ limit. For the chiral interaction r_0 reaches remarkable values of more than 50%. In case of a collective mode such a q behavior of r_0 can be expected, since low q correspond to large wavelengths, where the virtual photon “sees” the nucleus as a whole. The table also shows that r_1 is always smaller than r_0 . This indicates a non negligible high-energy strength and also shows that the argumentation against the collectivity which relies on r_1 is based on shaky ground.

Rather than considering its integral properties the study of the strength distribution itself

is much more informative. As already mentioned above, in [37] we had computed both the InISM background distribution and the integrated 0_R^+ strength (though not the strength distribution of the resonance itself). There we focused on E_R and $|\mathcal{F}_{\mathcal{M}}(q)|^2$, which resulted to be largely model dependent and considerably higher than existing data. Here, in Fig. 2 we show $S_{\mathcal{M}}(q, \omega)/m_0(q)$ at constant q -values for the chiral interaction, assuming for the 0_R^+

TABLE I. Transition form factor $|F_{\mathcal{M}}(q)|^2$ and the zero-th and first moment of the strength distribution for the InISM operator $\mathcal{M}(q)$. Also listed are the corresponding ratios r_0 and r_1 . For each q the upper and lower lines refer to N³LO+N²LO and AV18+UIX forces, respectively.

q [$\frac{\text{MeV}}{c}$]	$ F_{\mathcal{M}}(q) ^2$	m_0	m_1 [MeV]	r_0 %	r_1 %
50	0.00034	0.00063	0.021	53	34
	0.00024	0.00064	0.018	38	28
100	0.0042	0.0085	0.262	50	34
	0.0031	0.0086	0.258	37	25
200	0.0248	0.0683	2.42	36	22
	0.0190	0.0710	2.48	27	16
300	0.0297	0.129	5.89	23	11
	0.0242	0.139	6.33	17	8
400	0.0154	0.126	8.43	12	4
	0.0141	0.143	9.39	10	3

a Lorentzian with the experimental width of 270 keV [8]. It is evident that the spectrum is completely dominated by the resonance peak. A similar result is obtained for the AV18+UIX potential even though the ratio resonance-background is somewhat smaller. In Fig. 2 the peak becomes less pronounced with growing q , in favor of an increasing background (see insets of Fig. 2). The more pronounced dominance at low q can be understood if one considers that \mathcal{M}^{LW} depends only on the collective variable ρ^2 . This is not the case for $\mathcal{M}(q)$ in Eq.(2), where the non-collective coordinates play a growing role with increasing q .

Here we would like to draw the attention to an interesting similarity between two very different physical systems: the evolution with q of the monopole spectrum of this light system resembles that of the dynamical structure factor of an alkali metal (almost free electron gas), where the plasmon (dipole) collective excitation is established at low q (see e.g. [40]).

Now we turn to another aspect related to the breathing mode, namely the nuclear (in)-compressibility and its relation to m_{-1}^{LW} . In [29] it is shown that a monopole perturbation $V_P = \lambda \sum_i r_i^2$ induces a change of the radius proportional to m_{-1}^{LW} ($\lim_{\lambda \rightarrow 0} \delta \langle r^2 \rangle / \lambda =$

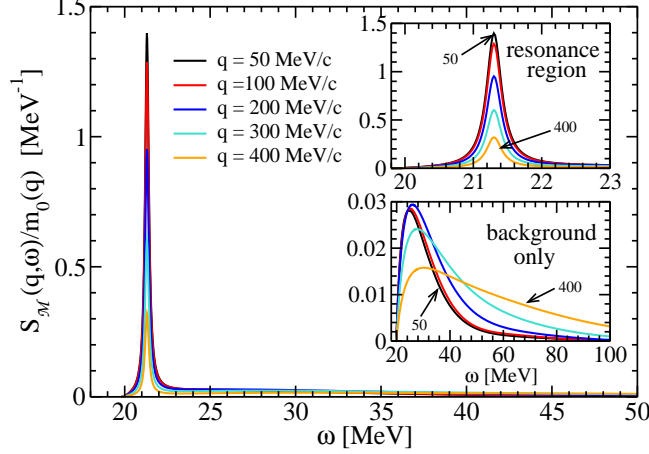


FIG. 2. (Color online) $S_{\mathcal{M}}(q, \omega)/m_0(q)$ for various fixed q . In the insets the strength in the resonance region and the background contribution.

$-2 m_{-1}^{\text{LW}}$). Therefore m_{-1}^{LW} serves to define the nuclear incompressibility [41] $K_A^I = A \langle r^2 \rangle^2 / (2m_{-1})$. We have calculated m_{-1}^{LW} summing the inverse-energy-weighted resonance strength to the corresponding integral of the background contribution. For the chiral interaction and for AV18+UIX we find $m_{-1}^{\text{LW}} = 0.259$ and $0.236 \text{ fm}^4 \text{ MeV}^{-1}$ (resonance strength contribution: 64% and 45%) and $\langle r^2 \rangle = 2.146$ and 2.051 fm^2 , respectively. These values lead to $K_4^I = 36 \text{ MeV}$ for both potentials. Another definition of the nuclear incompressibility in terms of the resonance energy $K_A^{II} = E_R^2 M \langle r^2 \rangle$ [41] is used in the literature. This is equivalent to K_A^I if one uses the sum rule estimate $E_R = E_R^{\text{SR}} = \sqrt{m_1^{\text{FEWSR}}/m_{-1}}$. For N³LO+N²LO and AV18+UIX we find $E_R = 21.25$ and 21.06 MeV , therefore $K_4^{II} = 23$ and 22 MeV , respectively. The reason why K_4^I differs from K_4^{II} is due to the discrepancy between the values of E_R and the higher sum rule estimates $E_R^{\text{SR}} = 26.2$ and 26.8 MeV , caused by the background contributions. All these values of the incompressibility are much smaller than the nuclear matter estimate ($230 \pm 40 \text{ MeV}$ [4]), showing the extreme softness of the α -particle. Such low K_4 are in line with a recent parametrization for nuclei with $A > 10$ [42]. Extending the fit to ${}^4\text{He}$ one obtains $K_4 \simeq 0$, because of the large surface contribution.

Summary. We have investigated the InISM strength of ${}^4\text{He}$ and the corresponding sum rules within a few-body *ab initio* approach, employing realistic nuclear forces (chiral and phenomenological ones). For the 0_R^+ we find properties that can also be attributed to a collective breathing mode: (i) the transition density changes sign at about the ${}^4\text{He}$ radius; (ii) for low momentum transfer q , where the excitation operator can be expressed in terms

of the collective hyperradius, the transition strength to 0_R^+ is large and exhausts between about 40% (phenomenological force) and 50% (chiral force) of the non energy-weighted sum rule and (iii) the resonance dominates the continuum spectrum completely over a very low and extended background.

Moreover, we observe a very interesting similarity in the evolution with q of the spectrum, between our results and the plasmon collective excitation spectrum of an alkali metal. Finally, the inverse energy weighted SR allows to give the incompressibility K_A of ^4He predicted by the modern realistic potentials: $22 \leq K_A \leq 36$ MeV.

A final clarification of the collectivity issue can only come from experiment. Since the present criterion based on the FEWSR is not really appropriate, it would be necessary to determine the strength distribution at low q , where existing electron scattering data are scarce and limited to the resonance strength. Interesting complementary information could come from α scattering.

W.L and G.O. would like to thank E. Lipparini, S. Stringari and F. Pederiva for helpful discussions. This work was supported in parts by the Natural Sciences and Engineering Research Council (NSERC), the National Research Council of Canada, the Israel Science Foundation (Grant number 954/09), and the Pazi Fund.

-
- [1] D.H. Youngblood, Y.-W. Lui, Krishichayan, J. Button, M. R. Anders, M. L. Gorelik, M. H. Urin, and S. Shlomo, Phys. Rev. C **88**, 021301 (2013).
 - [2] J.W. Abraham and M. Bonitz, Contrib. Plasma Phys. **54**, 27 (2014).
 - [3] E. Khan, J. Margueron, G. Colò, K. Hagino, and H. Sagawa, Phys. Rev. C **82**, 024322 (2010).
 - [4] E. Khan, J. Margueron, and I. Vidana, Phys. Rev. Lett. **109**, 092501 (2012).
 - [5] R.F. Frosch, R.E. Rand, M.R. Yearian, H. Crannell, and L.R. Suelzle, Phys. Lett. **19**, 155 (1965); R.F. Frosch, R.E. Rand, H. Crannell, J.S. McCarthy, L.R. Suelzle, and M.R. Yearian Nucl. Phys. A **110**, 657 (1968).
 - [6] C Werntz and H. Überall, Phys. Rev. **149**, 762 (1966).
 - [7] W. Leidemann and G. Orlandini, Progr. Part. Nucl. Phys. **68**, 158 (2013).
 - [8] Th. Walcher, Phys. Lett. B, **31**, 442 (1970).
 - [9] D.L. Hill and J.A. Wheeler, Phys. Rev. **89**, 1102 (1953).

- [10] J.J. Griffin and J.A. Wheeler, Phys. Rev. **108**, 311 (1957).
- [11] E. Caurier, B. Bourotte-Bilwes and Y. Abgrall, Phys. Lett. B **44**, 411 (1973).
- [12] Y. Abgrall and E. Caurier, Phys. Lett. B **56**, 229 (1975).
- [13] B. Schwesinger, Phys. Lett. B **103**, 182 (1981).
- [14] H. Liu and L. Zamick, Phys. Rev. C **33**, 1093 (1986).
- [15] G. Köbschall, C. Ottermann, K. Maurer, K. Röhrich, Vh. Schmitt and V.H. Walther, Nucl. Phys. A, **405**, 648 (1983).
- [16] V.S. Vasilevskii and I.Yu Rybkin, Sov. J. Nucl. Phys. **46**, 220 (1987).
- [17] V.S. Vasilevskii, T.P. Kovalenko, and G.F. Filippov, Sov. J. Nucl. Phys. **48**, 217 (1988).
- [18] W. Zickendraht, J. Math. Phys. **12**, 1663 (1971).
- [19] D. J. Rowe, and G. Rosensteel, Ann. Phys. **126**, 198 (1980).
- [20] D. Halderson, Ming Yu, and Jiang Yu, Phys. Rev. C **39**, 336 (1989).
- [21] E. Hiyama, B.F. Gibson, and M. Kamimura, Phys. Rev. C **70**, 031001R (2004).
- [22] B.S. Pudliner, V. R. Pandharipande, J. Carlson, S.C. Pieper, and R.B. Wiringa, Phys. Rev. C **56**, 1720 (1997).
- [23] R.A. Ferrell, Phys. Rev. **107**, 1631 (1957).
- [24] D.R. Entem and R. Machleidt, Phys. Rev. C **68**, 041001 (2003).
- [25] P. Navrátil, Few Body Syst., **41**, 117 (2007).
- [26] R.B. Wiringa, V.G.J. Stoks, and R. Schiavilla, Phys. Rev. C **51**, 38 (1995).
- [27] B.S. Pudliner, V.R. Pandharipande, J. Carlson, and R.B. Wiringa, Phys. Rev. Lett. **74**, 4396 (1995).
- [28] D.J. Rowe, "Nuclear Collective Motion", Methuen & Co. LTD., London, 1970.
- [29] O. Bohigas, A.M. Lane, and J. Martorell, Phys. Rep. **51**, 267 (1979).
- [30] E. Lipparini and S. Stringari, Phys. Rep. **175**, 104 (1989).
- [31] G. Orlandini and M. Traini, Rep. Prog. Phys. **54**, 257 (1991).
- [32] E.I. Kao and S. Fallieros, Phys. Rev. Lett. **25**, 827 (1970).
- [33] J. V. Noble, Ann. of Phys. (N.Y.) **67**, 98 (1971).
- [34] D. Gazit, N. Barnea, S. Bacca, W. Leidemann, and G. Orlandini, Phys. Rev. C **74**, 061001R (2006).
- [35] N. Barnea, W. Leidemann, and G. Orlandini, Phys. Rev. C **61**, 054001 (2000).
- [36] N. Barnea, W. Leidemann, and G. Orlandini, Nucl. Phys. **A693**, 565 (2001).

- [37] S. Bacca, N. Barnea, W. Leidemann, and G. Orlandini, Phys. Rev. Lett. **110**, 042503 (2013).
- [38] V.D. Efros, W. Leidemann, and G. Orlandini, Phys. Lett. B **338**, 130 (1994).
- [39] V.D. Efros, W. Leidemann, G. Orlandini, and N. Barnea, J. Phys. G **34**, R459 (2007).
- [40] A. vom Felde, J. Sprosser-Prou, and J. Fink, Phys. Rev. B **40**, 10181 (1989).
- [41] J.P. Blaizot, Phys. Rep. **64**, 171 (1980).
- [42] P. Veselý, J. Toivanen, B.G. Carlsson, J. Dobaczewski, N. Michel, and A. Pastore, Phys. Rev. C **86**, 024303 (2012).